TRINITY COLLEGE

Year 12 Mathematics Specialist
Test 2   2019

Section 1   Calculator Free
2D and 3D Vectors

STUDENT'S NAME   _____

DATE: Friday 5 April   **TIME:** 25 minutes   **MARKS**: 25

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser, formula page

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1.   (3 marks)

Given $\underset{\sim}{a} = \begin{pmatrix} -8 \\ 4 \\ 16 \end{pmatrix}$ and $\underset{\sim}{b} = \begin{pmatrix} 3 \\ 3 \\ 3 \end{pmatrix}$, determine a vector in the direction of $\underset{\sim}{a}$ with magnitude $\left| \underset{\sim}{b} \right|$

2.   (7 marks)

A plane contains the point $(1,2,3)$ and the vectors $\underset{\sim}{a} = \begin{pmatrix} -1 \\ -8 \\ 3 \end{pmatrix}$ and $\underset{\sim}{b} = \begin{pmatrix} 1 \\ -5 \\ 2 \end{pmatrix}$

(a)   Determine a normal to the plane.   [2]

(b)   Determine the vector equation of the plane in scalar form.   [2]

(c)   The line $\underset{\sim}{r} = \begin{pmatrix} 10 \\ 3 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$ intersects the plane. Determine the point of intersection. [3]
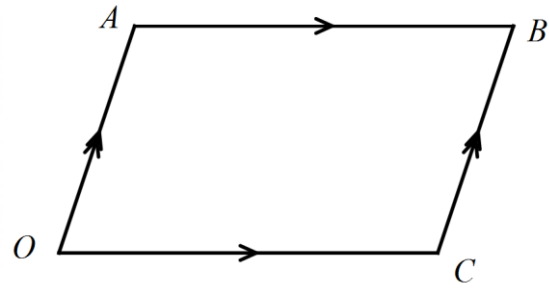
3.   (6 marks)

A parallelogram has the following information:

$$\overrightarrow{OA} = 5\underset{\sim}{i} + 5\underset{\sim}{j}$$

$$\overrightarrow{OC} = 6\underset{\sim}{i}$$

Where O is the origin.

(a)   Determine the position vector of B [2]

(b)   Determine the area of the parallelogram   [4]

4. (9 marks)

(a) Consider A, B and C with position vectors $\begin{pmatrix} 4 \\ -3 \\ 5 \end{pmatrix}$, $\begin{pmatrix} 6 \\ 7 \\ 9 \end{pmatrix}$ and $\begin{pmatrix} 7 \\ 2 \\ 6 \end{pmatrix}$ respectively. Show that these points form the vertices of an isosceles triangle.

[6]

(b) Determine the Cartesian equation of the plane that contains the isosceles triangle. [3]

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Section 2   Calculator Assumed
2D and 3D Vectors

STUDENT'S NAME   _____

DATE: Friday 5 April   **TIME:** 25 minutes   **MARKS**: 25

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser, formula page
Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this
                          assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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5.   (5 marks)

Two particles A and B have initial position vectors of $-73\underset{\sim}{i} + 242\underset{\sim}{j} - 476\underset{\sim}{k}$ and
$199\underset{\sim}{i} - 30\underset{\sim}{j} + 170\underset{\sim}{k}$ respectively and their velocity vectors are $4\underset{\sim}{i} - 13\underset{\sim}{j} + 29\underset{\sim}{k}$ and $-12\underset{\sim}{i} + 3\underset{\sim}{j} - 9\underset{\sim}{k}$
respectively. All units are S.I. units (meters and seconds).

(a)   Determine the position vector where the two paths intersect.   [3]

(b)   Do the two particles collide? Explain.   [2]

6.   (8 marks)

A and B have position vectors of $\begin{pmatrix} 2 \\ -6 \end{pmatrix}$ and $\begin{pmatrix} -3 \\ -5 \end{pmatrix}$ respectively. A line passes through the two points.

(a)   Determine the vector equation of the line.   [2]

(b)   Determine the parametric equations of the line.   [2]

(c)   Determine the Cartesian equation of the line.   [2]

(d)   Determine the angle the line makes with the y-axis.   [2]

7.   (7 marks)

A sphere has the vector equation $\left| \underset{\sim}{r} - \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \right| = 5$

(a)   Determine where the line $\underset{\sim}{r} = \begin{pmatrix} 1 \\ -26 \\ 24 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 4 \\ -3 \end{pmatrix}$ intersects the sphere.   [4]

(b)   The point $\begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix}$ lies on the surface of the sphere. Determine the equation of a line that

is tangent to the sphere at this point.   [3]

8. (5 marks)

The points $A\,(1,-1,3)$, $B\,(4,1,-2)$, $C\,(-1,-1,1)$ and $D\,(1,1,1)$ all lie on the surface of a sphere.

(a) Determine the centre and radius of the sphere. [3]

(b) Determine the equation of the plane that is tangent to the sphere at the point $D\,(1,1,1)$

[2]